The Value of Life across Generations: Fertility and Parental Investment*

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Abstract

We study survival valuation in a quantitative dynastic life-cycle model of heterogeneous agents choosing fertility and parental investment. Calibrated to the pre-baby-boom United States, the model generates a substantial dynastic premium over own-life valuation, amplified by parental investments. Removing earnings risk raises fertility more when descendants also benefit. Holding current fertility fixed, the premium likewise responds more strongly to risk removal across generations. Declining maternal mortality and improved old-age survival both raise fertility. The premium changes little under the former and rises under the latter. These findings link survival valuation to family decisions and descendants' economic and mortality conditions.

Keywords: Value of a Statistical Life, Dynastic Value of Life, Mortality Risk, Earnings Risk, Fertility, Parental Investments, Intergenerational Transmission

JEL classifications: J17, J13, E24, D15, D64

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1 Introduction

Improvements in survival are an important source of economic welfare that conventional measures of income growth do not fully capture. Murphy and Topel (2006) demonstrate the substantial economic value of improvements in health and longevity, while Jones and Klenow (2016) incorporate mortality into welfare comparisons across countries and over time.[1] These studies motivate a closer examination of what is captured by households' willingness to pay for lower mortality risk.

For parents who care about their descendants, the benefits of a lasting mortality improvement can extend beyond their own lives. The value parents assign to descendants' survival depends on the lifetime utility those descendants can expect, which is shaped by inherited resources and future earnings uncertainty. Longer lives can also reshape the quantity-quality trade-off, changing family size and investment per child and thereby affecting parents' willingness to pay for improved survival. How do fertility and parental investment shape dynastic survival valuation, and how does it vary with mortality conditions and earnings risk across generations?

We address these questions in a quantitative dynastic life-cycle model in which heterogeneous agents choose fertility, saving, education spending, and inter-vivos transfers. Following Becker and Barro (1988) and Barro and Becker (1989), parents value their own lifetime utility and the welfare of their descendants. Households differ in inherited assets, earnings capacity, ability, and altruism, and face earnings risk and borrowing constraints, as in a standard incomplete-markets framework (Aiyagari 1994). Children require parental time and financial resources, and childbirth exposes mothers to mortality risk. Education spending and transfers shape the economic circumstances inherited by the next generation. These investments also use resources that could otherwise finance parental consumption, linking the quantity-quality trade-off of Becker and Lewis (1973) to both the benefits of mortality improvements and the marginal utility of the resources used to pay for them.

We distinguish the life-cycle value of a statistical life (LVSL), which captures the own-life component of survival valuation, from the dynastic value of a statistical life (DVSL), which also includes parents' valuation of survival benefits accruing to their descendants. Our

[1] More recently, Adhami et al. (2026) quantify how population growth contributes to welfare growth when additional lives have positive value, and Trammell and Jones (2026) show how changes in the value of a statistical life can inform the measurement of living standards.

benchmark DVSL concerns a common improvement in young-adult survival across generations, expressed per unit of improvement in the current parent's survival probability. The difference between DVSL and LVSL is the *dynastic premium*. Both measures are evaluated from the perspective of the current parent.[2]

We calibrate the baseline economy using U.S. mortality conditions in 1940 and fertility outcomes of women born in 1906–1910, together with moments on income, intergenerational persistence, saving, and parental investment. With mean young-adult LVSL anchored by a $2 million benchmark in 2000 dollars, the calibrated model generates a mean LVSL of approximately $2.06 million and a mean DVSL of $2.98 million. The implied dynastic premium is $0.92 million, or approximately 31% of DVSL. Descendants' survival benefits therefore account for a substantial share of young parents' valuation of the common mortality improvement.

We further decompose the dynastic premium into a continuation component and contributions associated with inter-vivos transfers and parental education spending. The continuation component evaluates descendants' survival benefits at the child's inherited state when the current parent provides neither a financial transfer nor discretionary education spending. The child nevertheless retains the human capital generated by the baseline education input. A Shapley decomposition allocates the remaining premium between transfers and education, accounting for their interaction. Continuation accounts for approximately 79% of the premium, education for 17%, and transfers for 4%. Most of the premium therefore remains at the reference inherited state, while parental investments increase it further.

Using the calibrated baseline economy, we first use earnings-risk removal as a complementary mechanism exercise: holding mortality conditions fixed, we vary the economic uncertainty facing parents and descendants to examine how their economic prospects shape family choices and survival valuation. Specifically, we remove the earnings shock while preserving its mean multiplier, distinguishing risk removal for current parents alone from risk removal extending to descendants.[3] Removing parents' risk mainly raises the own-life component of survival valuation and leaves the dynastic premium almost unchanged. When risk removal extends across generations, the premium rises by approximately 3.3%, primarily through its

[2] In other words, DVSL incorporates the value the parent assigns to benefits received by descendants; it is not the sum of willingness to pay across different generations.

[3] In the counterfactual comparisons, we evaluate survival valuation measures at each current parent's baseline fertility, while allowing other current choices and descendants' decisions to adjust. Family-choice outcomes, by contrast, allow current fertility to change.

continuation component. Fertility also responds more strongly, while education spending per child falls and financial transfers remain nearly unchanged. These results show how descendants' economic prospects affect both family formation and the value parents assign to improved survival, even without a change in mortality parameters.

We then examine how mortality improvements affect the quantity-quality trade-off through two historical counterfactuals that update, separately, maternal mortality and old-age survival to their values one model period (20 years) after the baseline. These changes operate through different margins. Lower maternal mortality reduces a direct cost of childbearing, while improved old-age survival extends the expected lives of parents and their descendants. We recompute household decision rules under each change and evaluate the resulting outcomes using the baseline distribution of household states.[4]

We find that fertility increases by approximately 2.3% following the decline in maternal mortality and by 1.2% following the improvement in old-age survival. Both changes reduce childlessness. Total investment per child falls modestly under lower maternal mortality and remains approximately unchanged under improved old-age survival. The valuation responses differ more clearly: conditional on baseline fertility, the dynastic premium remains almost unchanged under lower maternal mortality but rises by approximately 2.1% under improved old-age survival, mainly through its continuation component. Thus, a mortality improvement that induces a larger fertility response need not generate a larger increase in the dynastic premium.[5]

Our analysis builds on the life-cycle literature on the valuation of mortality risk and longevity. Arthur (1981), Rosen (1988), and Ehrlich and Chuma (1990) develop life-cycle approaches in which the value of survival or life extension depends on age, resources, and consumption opportunities. Murphy and Topel (2006) provide a quantitative framework for measuring the economic value of improvements in health and longevity, while Hall and Jones (2007) connect the value of life to income and the demand for health spending.[6] We bring these valuation questions into a model of heterogeneous agents with endogenous fertility

[4]These exercises therefore isolate household responses to historically relevant mortality changes rather than providing a full transition accounting of the baby boom.

[5]If we instead calculate LVSL and DVSL using the fertility chosen under each counterfactual, both valuations are lower than in the baseline-fertility comparisons, as larger families reduce parental consumption and raise its marginal utility. The dynastic premium nevertheless remains positive.

[6]More recently, Bauer, Lakdawalla, and Reif (2025) show how health shocks and their financial consequences affect mortality valuation through changes in both expected lifetime utility and consumption.

and parental investments.[7] Birchenall and Soares (2009) incorporate altruism and endogenous fertility into mortality valuation, including parents' willingness to pay for reductions in child mortality and benefits accruing to future generations. Our focus is adult survival across generations in a quantitative model of heterogeneous agents with borrowing constraints, endogenous fertility, education spending, and inter-vivos transfers. This framework allows us to examine how family circumstances shape the quantity-quality trade-off and the dynastic premium.

The modeling framework draws on the literature on endogenous fertility and dynastic preferences. Becker and Lewis (1973) formalize the interaction between the quantity and quality of children, while Becker and Barro (1988) and Barro and Becker (1989) link fertility to parental altruism in dynastic models. Doepke (2005) examines the fertility implications of declining child mortality within this class of models. de La Croix and Doepke (2003) show how differential fertility and education connect inequality to growth, and Cordoba, Liu, and Ripoll (2016) examine fertility, social mobility, and long-run inequality in a heterogeneous-household setting.[8] Daruich and Kozlowski (2020) incorporate endogenous fertility, education, and family transfers into a heterogeneous-agent life-cycle model to study intergenerational mobility. Zhou (2025) examines the macroeconomic consequences of family policies, emphasizing the quantity-quality trade-off. We draw on this literature to model fertility and intergenerational investments jointly, and examine how these choices shape mortality valuation and responses to improved survival.

Our framework also relates to quantitative heterogeneous-agent models of parental investment and intergenerational transmission. Restuccia and Urrutia (2004), Lee and Seshadri (2019), Caucutt and Lochner (2020), and Yum (2023) study how parental investments and family resources shape the persistence of economic status. Fuchs-Schündeln et al. (2022), Jang and Yum (2024), Daruich (2026), Bellue and Mahler (2026) and Gu and Zhang (2026) examine the aggregate and intergenerational consequences of changes in education provision and early childhood investment. We study how these family links shape mortality valuation: parental investments affect descendants' opportunities and the value parents assign to their survival, while using resources that could finance current consumption. Endogenous fertility

[7] Our analysis is also complementary to Huetsch, Krueger, and Ludwig (2026), who study medical expansion and longevity through endogenous directed technical change. We take mortality changes as given and examine their valuation and consequences for family decisions.

[8] Kim, Tertilt, and Yum (2024) develop a quantitative model of endogenous fertility and parental education spending to study status externalities and low fertility. Their model does not feature dynastic utility. We adapt elements of its family-decision structure and embed them in a dynastic framework with mortality risk.

allows mortality improvements to change both family size and investment per child.

The paper also speaks to the empirical VSL and benefit-transfer literature. Following the early labor-market contribution of Thaler and Rosen (1976), a large literature estimates willingness to pay for mortality-risk reductions; see Kniesner and Viscusi (2019). Our analysis suggests that the portability of these estimates across applications may depend on who benefits from the mortality improvement and whether its benefits extend across generations. The dynastic premium provides a structural framework for understanding how intergenerational benefits enter willingness to pay and for interpreting VSL estimates across mortality contexts.

The remainder of the paper proceeds as follows. Section 2 presents the model and defines the valuation concepts. Section 3 describes the calibration. Section 4 presents baseline valuations and decomposes the dynastic premium. Section 5 examines earnings risk and dynastic valuation, and Section 6 examines the historical mortality changes. Section 7 examines sensitivity to consumption curvature, and Section 8 concludes.

2 Model and the Dynastic Value of Statistical Life

We build a four-period dynastic life-cycle model with mortality risk. Households choose fertility, saving, education spending, and inter-vivos transfers while facing earnings risk and borrowing constraints. Parents value their own lifetime utility and the welfare of their descendants, so family choices determine both the resources inherited by the next generation and the value parents assign to its survival. This structure allows us to define and distinguish the life-cycle and dynastic values of a statistical life.

2.1 Demographics, heterogeneity, and timing

Individuals live for up to four adult periods, indexed by $j = 1, \ldots, 4$. Each period lasts 20 years and corresponds to ages 20–39, 40–59, 60–79, and 80–99, respectively. Individuals work in the first two periods and receive retirement income in the last two. We use a symmetric per-adult representation of families, with each decision maker corresponding to one adult parent. Consumption, earnings, and assets are measured per adult, whereas fertility n

denotes the actual number of children per couple. We use female mortality to describe the modeled parent's survival.

A young individual enters adulthood with assets a_1, earnings capacity e_1, and ability ϕ. Initial assets are determined by transfers from its parents, while earnings capacity reflects its ability and the education investment received during childhood. Ability also governs the conditional distribution of offspring ability, denoted by $F(\phi_k|\phi)$, where the subscript k refers to the next generation.

At the beginning of the first period, the individual observes its altruism type α and chooses fertility from the finite set $\{0, 1, \ldots, \bar{n}\}$ where $\bar{n} = 4$. Fertility is chosen before offspring ability ϕ_k is observed. After observing ϕ_k, the parent chooses education spending per child s, inter-vivos transfers per child a_k, and saving for its own second period a_2. The model represents offspring by a common child state, with the same education spending and transfer for each child. In the second period, an earnings shock z is realized. This shock affects second-period earnings and the income on which retirement benefits are based.

Altruism is drawn independently across generations from a common distribution, independently of inherited states. Thus, a parent's altruism is known when fertility is chosen, whereas the altruism of the next generation is integrated out in its continuation value. Ability, in contrast, is transmitted across generations through $F(\phi_k|\phi)$. We specify the intergenerational ability process as

$$\log \phi_k = \rho_\phi \log \phi + \varepsilon_{\phi,k}, \qquad \varepsilon_{\phi,k} \sim N(0, \sigma_\phi^2). \tag{1}$$

Thus, ρ_ϕ governs intergenerational persistence and σ_ϕ is the standard deviation of the innovation to log ability.

2.2 Preferences and dynastic altruism

While alive, the household receives flow utility from consumption c

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} + b, \tag{2}$$

where $\sigma > 0$ governs consumption curvature and $1/\sigma$ is the elasticity of intertemporal substitution. The parameter $b > 0$ is a utility flow from being alive (Hall and Jones 2007).

Its level matters for survival valuation because the parent's own utility flow after death is normalized to zero. Future own-life utility is discounted by β per model period.

Parents also value the dynastic utility of their children (Becker and Barro 1988; Barro and Becker 1989). Conditional on realized types and choices, the objective takes the form

$$V_1 + \alpha g(n) W' - \delta \mathbf{1}_{\{n>0\}}, \tag{3}$$

where V_1 denotes the parent's own expected lifetime utility, W' is the child's dynastic continuation value, and $\delta \geq 0$ is a fixed utility cost of parenthood.[9] Preferences are additive between own lifetime utility and offspring dynastic utility. The altruism parameter $\alpha > 0$ and the fertility aggregator $g(n)$ determine the weight on the next generation.

We allow altruism to differ across households according to $\alpha \sim F_\alpha(\alpha)$ with $\mu_\alpha > 0$ and $\sigma_\alpha \geq 0$ controlling the location and dispersion of altruism, respectively. Altruism is drawn independently across households and generations. This heterogeneity allows households with similar resources to make different fertility and investment choices, which helps the model match the negative income-fertility relationship in the data.[10]

Following Cordoba, Liu, and Ripoll (2016), we use an exponential fertility aggregator and normalize the weight for a one-child family:

$$g(n) = \frac{1 - \exp(-\epsilon n)}{1 - \exp(-\epsilon)}, \qquad \epsilon > 0. \tag{4}$$

This specification satisfies $g(0) = 0$ and $g(1) = 1$. Consequently, α is the altruistic weight for a one-child family, while ϵ governs how rapidly the incremental weight declines with fertility.[11]

[9] The fixed utility cost captures nonpecuniary costs of entering parenthood, such as reduced flexibility, and helps match childlessness while preserving altruistic motives for parental investment. For related fixed costs of parenthood, see Gobbi (2013) and Baudin, De La Croix, and Gobbi (2015).

[10] Jones, Schoonbroodt, and Tertilt (2010) emphasize preference heterogeneity as a potential explanation for the observed negative income–fertility relationship.

[11] The continuous extension of g is increasing and concave, with fertility elasticity $\epsilon n/[\exp(\epsilon n) - 1]$ for $n > 0$. Unlike a power aggregator with a constant fertility elasticity, this elasticity falls with n. Additional children therefore contribute progressively less to the total altruistic weight. The aggregator is also bounded, which is useful when controlling the dynastic continuation weight in the numerical solution.

2.3 Income, child costs, and intergenerational transmission

We express the budget in per-adult units, while fertility n counts children per couple. Under the symmetric allocation of resources and child costs between adults, the first-period budget constraint is

$$c_1 + a_2 + \frac{n}{2}(s + a_k) = (1 - \lambda n)e_1 + (1 + r)a_1, \tag{5}$$

where r is the interest rate on the non-state-contingent assets a_j over a model period. Each child requires a fraction λ of each adult's first-period time. Thus, child-rearing reduces each adult's earnings by $\lambda n e_1$. Education spending s and financial transfers a_k are amounts per child. The couple's total monetary expenditure is therefore $n(s + a_k)$, of which each adult finances one-half. Consumption must be positive, and saving, education spending, and transfers are nonnegative.

Education spending determines the child's initial adult earnings capacity according to

$$e_k = A\phi_k(\theta + s)^\nu, \qquad A > 0, \quad \theta > 0, \quad 0 < \nu < 1, \tag{6}$$

similar to the human-capital production of de La Croix and Doepke (2003). The parameter A determines the scale of earnings capacity and ν governs the curvature of the technology. The input θ provides positive earnings capacity even in the absence of parental education spending. The child enters adulthood with assets a_k, earnings capacity e_k, and ability ϕ_k.[12] Parents therefore affect their children's opportunities through both financial transfers and human capital.

In the second period, earnings capacity before the realization of earnings risk is $e_2 = \zeta e_1$, where $\zeta > 1$ captures life-cycle wage growth (Lagakos et al. 2018). After first-period saving has been chosen, the household draws a log earnings shock (Huggett, Ventura, and Yaron 2011):

$$z \sim F_z(z), \qquad \mathbb{E}[z] = 0, \qquad \mathrm{Var}(z) = \sigma_z^2.$$

The shock is independent across households and generations and is realized at the beginning of the second period. Second-period earnings are then $\exp(z)e_2$. The shock is permanent in the sense that realized earnings capacity, $e_3 = \exp(z)e_2$, also determines retirement income.

[12] Under symmetric matching, both partners bring the same inherited assets to their new couple. Their combined assets are $2a_k$, so initial assets per adult remain a_k.

The remaining budget constraints are

$$c_2 + a_3 = \exp(z)e_2 + (1+r)a_2, \tag{7}$$

$$c_3 + a_4 = \mathcal{P}e_3 + (1+r)a_3, \tag{8}$$

$$c_4 = \mathcal{P}e_3 + (1+r)a_4, \tag{9}$$

where $\mathcal{P} \in (0,1)$ is the retirement-income replacement rate. Households are subject to borrowing constraints: $a_2, a_3, a_4 \geq 0$. We abstract from accidental bequests: assets held at death do not generate transfers to descendants in the model. Intergenerational financial transfers are instead chosen explicitly in the first period.

2.4 Mortality and the household problem

First-period survival depends on fertility:

$$\pi_{1,n} = \tilde{\pi}_1 \exp(-\kappa n), \qquad \kappa \geq 0, \tag{10}$$

where $\tilde{\pi}_1$ is survival in the absence of the mortality risk associated with childbirth and κ is the maternal mortality hazard per birth. Survival from period 2 to period 3 is π_2, and survival from period 3 to period 4 is π_3. These later survival probabilities do not depend on fertility. All individuals die after the fourth period (ages 80–99).

Survival probabilities weight the parent's own future utility. Dynastic altruism instead represents the parent's current concern for descendants' welfare, including outcomes occurring beyond its own lifetime. We assume that first-period education spending and transfers are implemented before the survival transition and that the child's inherited state does not otherwise depend on subsequent parental survival.[13]

We describe the household problem backward from the final period. The value in $j = 4$ is

$$V_4(a_4, e_3) = u(\mathcal{P}e_3 + (1+r)a_4)\,. \tag{11}$$

[13]The offspring continuation term is therefore not multiplied by the parent's survival probability. This specification abstracts from additional effects of parental death on children's care, resources, and human-capital formation.

In the third period, the household solves

$$V_3(a_3, e_3) = \max_{a_4 \geq 0} \{u(c_3) + \beta \pi_3 V_4(a_4, e_3)\}, \tag{12}$$

subject to equation (8). In the second period, after observing z, it solves

$$V_2(a_2, e_2, z) = \max_{a_3 \geq 0} \{u(c_2) + \beta \pi_2 V_3(a_3, \exp(z) e_2)\}, \tag{13}$$

subject to equation (7).

In the first period, conditional on fertility and realized child ability, the household solves

$$\begin{aligned} P(a_1, e_1, \phi_k, \alpha; n) = \max_{a_2, s, a_k \geq 0} \Big\{ & u(c_1) + \beta \pi_{1,n} \mathbb{E}_z[V_2(a_2, \zeta e_1, z)] \\ & + \alpha g(n) W(a_k, e_k, \phi_k) - \delta \mathbf{1}_{\{n>0\}} \Big\}, \end{aligned} \tag{14}$$

subject to equations (5) and (6). For a childless household, the offspring term is absent and we set $s = a_k = 0$.

The ex-ante dynastic value, before the altruism draw, is given by

$$W(a_1, e_1, \phi) = \mathbb{E}_\alpha \left[\max_{n \in \{0,1,\ldots,\bar{n}\}} \mathbb{E}_{\phi_k | \phi}[P(a_1, e_1, \phi_k, \alpha; n)] \right]. \tag{15}$$

The ordering of the expectations and maximization reflects the timing: parents choose fertility after observing altruism but before observing child ability, and choose investments after observing child ability. In a stationary environment, the same function W applies to each generation. We use a prime below when helpful to distinguish the offspring's value from that of the current parent.

2.5 Stationary Equilibrium

A stationary equilibrium consists of optimal household value and policy functions and an invariant distribution of young agents. This distribution is reproduced across generations by fertility choices, parental education spending and inter-vivos transfers, and the transmission of ability: households with more children contribute proportionately more weight to the next generation. The corresponding cross-sectional distribution over ages is generated by optimal

saving, earnings shocks, and survival, accounting for differences in cohort size. Stationarity refers to these normalized distributions and is consistent with non-zero population growth. Appendix A.1 provides the formal laws of motion and the equilibrium definition.

2.6 Life-cycle and dynastic values of statistical life

We express the marginal value of survival in units of the modeled parent's current consumption. Let x_j denote a one-time resource transfer to that parent, added to the period-j budget constraint. At an optimum, the marginal value of these resources in the household's objective is $u'(c_j) = c_j^{-\sigma}$. The direct contribution of a marginal increase in own survival to own-life utility is $\beta \mathbb{E}_j[V_{j+1}]$, since own utility upon death is zero. We therefore define the life-cycle value of statistical life (LVSL) as

$$LVSL_j \equiv \beta c_j^{\sigma} \mathbb{E}_j[V_{j+1}], \qquad j = 1, 2, 3, \tag{16}$$

where the expectation is conditional on information available at the time of valuation. At age 1, it integrates over the future earnings shock z. All quantities are evaluated at the choices made in the dynastic model, so parental investment and altruism can affect LVSL through consumption and saving even though LVSL values only the parent's own remaining life.

To define dynastic VSL (DVSL), consider a small permanent improvement h in the common first-period survival environment:

$$\pi_{1,n}(h) = (\tilde{\pi}_1 + h)\exp(-\kappa n), \qquad h = 0 \text{ in the reference environment.} \tag{17}$$

The change applies to the current household and all subsequent generations.[14] Holding fertility fixed locally, the change in the current parent's actual survival probability is $\partial \pi_{1,n}/\partial h = \exp(-\kappa n)$.

Consider a household after its altruism and child ability are known, and evaluate its conditional value at the fertility it has chosen. Write this optimized value as $P^*(h, x_1)$, suppress-

[14]The LVSL age profile reported later values an increase in each age group's own conditional survival probability π_j. By contrast, the common perturbation considered here improves young-adult survival across generations. For households already past young adulthood, it does not improve their own remaining survival probabilities, although they may value descendants' benefits. Their willingness to pay therefore cannot be normalized by an increase in their own survival probability, as in our definition of $DVSL_1$. It could instead be expressed per unit of the common young-adult survival perturbation. Alternatively, one could consider common survival improvements targeting other ages. We leave these extensions for future work.

ing its other arguments. Away from a fertility threshold and at points where the value is differentiable, the envelope theorem gives

$$\left.\frac{\partial P^*}{\partial h}\right|_{h=0} = \beta \exp(-\kappa n)\mathbb{E}_z[V_2] + \alpha g(n) \left.\frac{\partial W'(a_k, e_k, \phi_k; h)}{\partial h}\right|_{h=0}. \tag{18}$$

Here the derivative of W' holds the inherited child state fixed, while allowing descendants to optimize in the changed environment. The current parent's adjustments in saving, education, and transfers do not enter separately into the first-order welfare derivative, because these choices were optimal in the reference environment.

We define dynastic VSL per unit increase in the current parent's actual survival probability:

$$DVSL_1 \equiv \frac{\partial P^*/\partial h|_{h=0}}{\partial P^*/\partial x_1|_{h=0}} \left(\frac{\partial \pi_{1,n}}{\partial h}\right)^{-1} \tag{19}$$

$$= LVSL_1 + \alpha g(n) c_1^{\sigma} \exp(\kappa n) \left.\frac{\partial W'(a_k, e_k, \phi_k; h)}{\partial h}\right|_{h=0}. \tag{20}$$

The second term is the *dynastic premium*. It captures the value to the current household of descendants facing the same improved survival environment. For a childless household, this term is zero. For a small change dh, willingness to pay is approximately $DVSL_1 \exp(-\kappa n)dh$ in current consumption units.

This definition distinguishes the scope of the survival improvement from its units of measurement. Both LVSL and DVSL are expressed per unit change in current survival, but DVSL also includes benefits to future generations. Note that the valuation is conditional on realized types and expressed in each household's own consumption units; reported averages are formed after calculating these household-level values. Discrete fertility thresholds can generate kinks, so the local formulas apply on a given optimal fertility branch.

3 Quantitative Implementation

We calibrate the model to a U.S. baseline combining mortality conditions around 1940 with the fertility experience of women born in 1906–1910. The calibration provides a common starting point for studying how earnings risk and mortality conditions shape family choices and survival valuation. We first describe the historical counterparts of the model, then discuss externally assigned parameters and the joint calibration of the remaining parameters.

3.1 Historical baseline and empirical counterparts

We use the 1940 female period life table to describe baseline survival. Period mortality summarizes the age-specific mortality environment at a given date, without incorporating the subsequent mortality improvements experienced by an actual birth cohort. This distinction is useful for our counterfactual exercises, which change selected survival parameters relative to a common baseline. Female survival describes the modeled parent's mortality.

Fertility targets come from the 1906–1910 U.S. birth cohorts, observed at ages 40–44 in 1950, as reported by Baudin, De La Croix, and Gobbi (2015). These women provide a counterpart for family formation before the postwar baby boom, although their reproductive years are not strictly confined to the prewar period. We use their reported fertility as an approximation to completed fertility. We target mean completed fertility of 2.30 children per woman.[15]

We combine these historical demographic targets with standard moments of earnings, wealth, and intergenerational investment. These economic moments draw on the sources described below and are not restricted to 1940.

3.2 Externally set parameters

Table 1 reports the externally set parameters. We set consumption curvature to $\sigma = 1.5$ and the annual risk-free interest rate to 4%, so that the return over a model period is $r = 1.04^{20} - 1$. The age-earnings parameter is $\zeta = 1.30$, guided by Lagakos et al. (2018), and the retirement-income replacement rate is $\mathcal{P} = 0.40$. Following de La Croix and Doepke (2003), we set the curvature of the human-capital technology to $\nu = 0.635$.

Let l_x denote the number of survivors to exact age x in the life table's hypothetical cohort. The 1940 female period table gives conditional survival probabilities of $l_{40}/l_{20} = 0.9443$, $l_{60}/l_{40} = 0.8291$, and $l_{80}/l_{60} = 0.3701$. We use the latter two ratios directly for π_2 and π_3. Young-adult survival requires an adjustment because the observed ratio already includes maternal deaths, whereas the model specifies survival conditional on fertility as $\pi_{1,n} = \tilde{\pi}_1 \exp(-\kappa n)$. The 1940 maternal mortality rate of 376 deaths per 100,000 live births

[15] The childlessness rate is 0.203 and mean completed fertility conditional on motherhood is 2.88, implying unconditional mean completed fertility of 2.30. This is a cohort fertility target, rather than the period total fertility rate in 1940.

Table 1: Externally set parameters

Parameter	Value	Interpretation	Source or basis
σ	1.50	Consumption curvature	CRRA benchmark
$\tilde{\pi}_1$	0.9525	Young-adult survival absent maternal risk	1940 female period table, adjusted
π_2	0.8291	Survival from age 40 to 60	1940 female period table
π_3	0.3701	Survival from age 60 to 80	1940 female period table
r	$1.04^{20} - 1$	Interest rate per model period	4% annual rate
ζ	1.30	Age-earnings profile	Lagakos et al. (2018)
$\mathcal{P}$	0.40	Retirement replacement rate	Benchmark
κ	0.00377	Maternal mortality hazard per birth	U.S. Vital Statistics, 1940
ν	0.635	Human-capital curvature	de la Croix and Doepke (2003)
λ	0.184	Child-rearing time cost per child	Lifetime time-cost benchmark

Notes: One period is 20 years. Survival probabilities are based on SSA female period life-table ratios. The adjustment to $\tilde{\pi}_1$ and the mapping from lifetime time costs to λ are described in the text.

implies $\kappa = 0.00377$. Adjusting observed young-adult survival for maternal mortality using mean completed fertility, we set $\tilde{\pi}_1 = 0.9525$.[16]

Child-rearing time costs are incurred during the first working period. We set $\lambda = 0.184$ using a lifetime time-cost benchmark guided by Cordoba, Liu, and Ripoll (2016). For a reference household with two children, the time cost per child equals 15% of the present value of working-life earnings net of child-rearing time costs.[17]

3.3 Internally calibrated parameters and model fit

We jointly calibrate the remaining eleven parameters by minimizing an equally weighted sum of squared percentage deviations between the model-generated moments and their em-

[16] Specifically, we set $\kappa = -\log(1 - 376/100{,}000)$. Exact matching of average survival would require $\tilde{\pi}_1 = 0.9443/\mathbb{E}[\exp(-\kappa n)]$. We use the approximation $\tilde{\pi}_1 \approx 0.9443 \exp(\kappa \bar{n}^{\text{data}}) = 0.9525$, where $\bar{n}^{\text{data}} = 2.298$. Note that this does not impose an exact survival match under the model's endogenous fertility distribution.

[17] Abstracting from mortality and earnings risk, the mapping is

$$\frac{\lambda e_1}{(1-2\lambda)e_1 + \zeta e_1/(1+r)} = 0.15, \qquad \lambda = \frac{0.15[1+\zeta/(1+r)]}{1+2(0.15)} \approx 0.184.$$

The denominator deducts the time costs of both children and excludes retirement income. The calculation concerns parental time alone; education spending and inter-vivos transfers enter the household budget separately.

Table 2: Internally calibrated parameters and model fit

Parameter	Value	Target moment	Data	Model
b	10.78	Young-adult life-cycle VSL (millions of 2000 dollars)	2.00	2.06
μ_α	0.263	Inter-vivos transfers / GDP	0.020	0.020
β	0.584	Capital-output ratio, K/Y	0.150	0.148
ρ_ϕ	0.242	Intergenerational income elasticity	0.350	0.350
σ_ϕ	0.461	$\mathrm{Var}(\log y_1)$	0.550	0.548
σ_z	0.461	$\mathrm{Var}(\log y_2) - \mathrm{Var}(\log y_1)$	0.180	0.180
A	3.05	Average income (normalization)	1.00	0.985
δ	4.40	Childlessness rate	0.203	0.202
θ	0.029	Education expenditures / GDP	0.073	0.072
ϵ	0.671	Completed fertility, bottom income half	2.58	2.52
σ_α	0.336	Completed fertility, top income half	2.01	2.10

Notes: Parameters are calibrated jointly. Fertility targets include childless households. The bottom and top income groups each represent half of the population. Sources and the construction of the targets are discussed in the text. Reported values are rounded.

pirical counterparts reported in Table 2. The table associates each parameter with a particularly informative target, but these associations are not separate, one-parameter calibrations. Because endogenous fertility and parental investment shape the stationary distribution, simultaneously matching income dispersion, intergenerational persistence, and the income-fertility gradient imposes nontrivial joint restrictions on the calibration.

The utility flow from being alive, b, is disciplined by a target of \$2 million in 2000 dollars for mean young-adult life-cycle VSL. We interpret this benchmark as willingness to pay for a reduction in the modeled parent's own mortality risk and therefore map it to LVSL rather than to the valuation of a common survival improvement across generations.[18] We convert model valuations into 2000 dollars by dividing by baseline mean working-age income per adult and multiplying by $20 \times \$35{,}296$. The same baseline conversion applies in every counterfactual environment. The \$2 million target therefore corresponds to approximately 2.83 times mean income over a model period.

We construct a discrete approximation to the altruism distribution using Tauchen discretiza-

[18] Empirical own-risk valuations may also reflect concern for family members affected by the respondent's survival. The mapping to LVSL is therefore a calibration assumption, rather than an exact correspondence between the model and every empirical VSL estimate. Dynastic VSL and the dynastic premium are not targeted in the calibration.

tion with zero persistence.[19] Let t_i and p_i denote the resulting standard-normal nodes and probabilities. Altruism is given by

$$\alpha_i = \max\{0, \mu_\alpha(1 + \sigma_\alpha t_i)\}, \qquad \Pr(\alpha = \alpha_i) = p_i, \qquad i = 1, \ldots, N_\alpha. \tag{21}$$

Mean altruism, μ_α, is disciplined by inter-vivos transfers relative to GDP, with a target of 0.020 guided by McGarry (1999). The discount factor β is informed by the capital-output ratio. Measured relative to output over a 20-year period, the target of 0.150 corresponds to capital worth three years of annual output. The parameters ρ_ϕ and σ_ϕ govern persistence and innovation dispersion in the intergenerational ability process. We use an intergenerational income elasticity of 0.350, close to the U.S. family-income estimates of Chetty et al. (2014), together with a target of $\mathrm{Var}(\log y_1) = 0.550$ to discipline these parameters jointly. The subsequent increase in earnings dispersion over the life cycle, $\mathrm{Var}(\log y_2) - \mathrm{Var}(\log y_1)$, is used separately to discipline the second-period earnings shock. These inequality moments are based on Heathcote, Perri, and Violante (2010).

The productivity scale A normalizes mean income to one. The fixed utility cost of parenthood, δ, provides a separate margin for matching childlessness. The education-technology shift θ is informed by education spending relative to GDP, targeted at 0.073 following de La Croix and Doepke (2003). Because this shift affects the return to parental education spending, it also interacts with fertility and altruism in determining investment per child.

We use fertility differences by income to discipline the curvature of the fertility aggregator, ϵ, and altruism dispersion, σ_α. Since $g(1) = 1$, μ_α is the mean altruistic weight for a one-child family at the calibrated distribution.Variation in altruism allows households with similar resources to make different fertility and investment choices. To construct the fertility targets, we combine the aggregate cohort mean of 2.30 from Section 3.1 with a bottom-top income fertility gap of 0.57, based on Jones and Tertilt (2008).[20]

The model closely reproduces the targeted intergenerational income elasticity and earnings-dispersion moments and provides a good overall fit to transfers, saving, education spending, and childlessness. Mean young-adult LVSL is approximately \$2.06 million, compared with the \$2 million target. Completed fertility is 2.52 in the lower-income half and 2.10 in the

[19] We use $N_\alpha = 10$ nodes spanning $[-2.5, 2.5]$, with probabilities given by standard-normal midpoint-bin masses and the tails assigned to the endpoint nodes.

[20] Centering this gap around the aggregate mean gives $n_L^{\text{data}} = \bar{n}^{\text{data}} + 0.57/2 = 2.58$ and $n_H^{\text{data}} = \bar{n}^{\text{data}} - 0.57/2 = 2.01$.

Table 3: Baseline value of statistical life

	Value ($1,000)	Share of $DVSL_1$
Life-cycle VSL, $LVSL_1$	2,061	69.2%
Dynastic premium, $DVSL_1 - LVSL_1$	916	30.8%
Dynastic VSL, $DVSL_1$	2,977	100.0%

upper-income half. The model therefore captures the negative income-fertility relationship reasonably well. Mean completed fertility is 2.31, close to 2.30 in the data. First-period log-income variance is 0.548 and its subsequent increase is 0.180, compared with targets of 0.550 and 0.180, respectively.

4 Baseline Valuation and the Dynastic Premium

We begin by comparing the own-life and dynastic values of a common survival improvement in the baseline economy. We then decompose the resulting dynastic premium into a continuation component and the contributions of parental financial transfers and human-capital investment. Finally, we examine how the life-cycle value of survival varies with age.

The magnitude of the dynastic premium Table 3 reports the population means of the life-cycle and dynastic values of statistical life at age 1. All values are expressed in 2000 dollars. As defined in Section 2.6, both objects are measured per unit increase in the current parent's survival probability. The distinction is that $LVSL_1$ values only the parent's own remaining lifetime utility, whereas $DVSL_1$ also incorporates the value that the parent assigns to the survival benefits received by future generations under a common permanent mortality improvement.

Mean dynastic VSL is approximately $2.98 million, compared with a life-cycle VSL of $2.06 million. The dynastic premium is therefore approximately $0.92 million and accounts for 30.8% of dynastic VSL. Equivalently, incorporating descendants' survival benefits raises the current parent's willingness to pay by approximately 44% relative to the own-life component. The own-life valuation is anchored by the calibration, whereas the additional dynastic valuation is a model implication.

Table 4: Decomposition of the baseline dynastic premium

Component	Value ($1,000)	Share
Continuation, C	719	78.5%
Inter-vivos transfers, S_a	38	4.2%
Human capital, S_e	158	17.3%
Total dynastic premium	916	100.0%

Notes: Values are in thousands of 2000 dollars. Components are calculated at the individual parent level and averaged using baseline population weights. Shares are ratios of mean components to the mean premium.

The premium reflects more than the survival benefit received by an existing child. The common improvement also reaches the child's descendants and subsequent generations, whose benefits enter through the dynastic continuation value. Its magnitude nevertheless depends on the current parent's altruism, fertility, marginal utility of consumption, and the economic state inherited by the child. We next ask how much of the premium remains without the current parent's financial transfers and discretionary education spending, and how much these investments add.

Decomposing the dynastic premium We separate the premium into dynastic continuation and the contributions of financial transfers and human-capital investment. Throughout this calculation, we hold the current parent's fertility, consumption, and altruism, as well as child ability, fixed.

The continuation component, C, evaluates descendants' survival benefits at an inherited state without financial transfers or discretionary education spending by the current parent. The child retains the baseline human-capital input θ, and descendants optimize conditional on their inherited circumstances. We attribute the remaining premium to transfers, S_a, and human capital, S_e, using a Shapley decomposition that averages their marginal contributions over the two possible orderings. By construction,

$$DVSL_1 - LVSL_1 = C + S_a + S_e. \tag{22}$$

Appendix A.2 provides the formal definitions and construction.

Table 4 shows that continuation accounts for 78.5% of the premium. Most of the premium therefore remains when evaluated without the current parent's financial transfers or discre-

Table 5: Life-cycle value of statistical life by age

Model period	Age range	Mean $LVSL_j$ ($1,000)
$j = 1$	20–39	2,061
$j = 2$	40–59	8,130
$j = 3$	60–79	6,873

tionary education spending. These investments account for the remaining 21.5%, with human capital contributing 17.3% and inter-vivos transfers 4.2%.

This is an accounting decomposition, not a measure of investment responses to mortality improvements. The investment contributions capture how inherited resources affect descendants' choices and marginal survival valuation, rather than simply the value of the resources provided. Behavioral responses are examined in Sections 5 and 6.

The life-cycle profile of the value of survival The age profile of VSL is theoretically ambiguous. Standard life-cycle models imply that a shorter remaining lifetime tends to reduce the value of survival, but changes in consumption, wealth, and marginal utility over the life cycle can offset this effect (Rosen 1988; Ehrlich and Chuma 1990).

The same forces operate in our model. Recall from equation (16) that

$$LVSL_j = \beta c_j^{\sigma} \mathbb{E}_j[V_{j+1}], \qquad j = 1, 2, 3. \tag{23}$$

The age profile reflects two components. First, the continuation value $\mathbb{E}_j[V_{j+1}]$ generally declines as the remaining lifetime shortens. Second, current consumption determines the conversion of utility into monetary units: higher consumption lowers marginal utility and raises willingness to pay for a given utility gain from survival. If consumption rises over part of the life cycle, this second component can offset the decline in continuation value. Consequently, VSL need not decline monotonically with age.

The hump-shaped age profile in Table 5 illustrates the interaction between the utility benefit of survival and its valuation in current consumption units. During the childbearing period, child-rearing time costs, education spending, inter-vivos transfers, and saving compete with current consumption, while borrowing constraints limit consumption smoothing. The resulting high marginal utility makes parents less willing to surrender current resources for a

given utility gain from survival. In the second period, direct child-related expenditures are no longer incurred and higher consumption increases the monetary valuation of survival. At older ages, however, the shorter remaining lifetime reduces continuation utility. The middle-age peak reflects the balance between these consumption and continuation-value forces.

This interpretation does not imply that $LVSL_1$ excludes later-life utility: V_2 already incorporates utility at all subsequent ages, weighted by discounting and survival. Rather, $LVSL_1$ values survival into period 2 in first-period consumption units, whereas $LVSL_2$ values survival into period 3 in second-period consumption units. The age profile therefore compares different survival margins and consumption circumstances. Appendix A.5 separates the consumption and continuation-value components underlying this profile.

5 Earnings Risk and Dynastic Valuation

Uninsured earnings risk affects consumption and self-insurance through saving (Aiyagari 1994). It can also shape family decisions: Clark and Lepinteur (2022) show that increased job insecurity reduced childbearing among employed workers in France. In a quantitative life-cycle model, Sommer (2016) examines how earnings uncertainty can contribute to delayed childbearing and smaller families. We use the calibrated model to examine how earnings uncertainty affects family choices and the value of survival, distinguishing the effects of removing risk for current parents alone from those of removing risk across generations. We hold mortality conditions fixed throughout these exercises and examine changes in mortality separately in Section 6.

5.1 Design of the experiments

We remove the second-period earnings shock while preserving the mean earnings multiplier. Specifically, we replace $\exp(z)$ with its baseline expectation $\bar{m}_z = \mathbb{E}[\exp(z)]$, calculated using the baseline discrete shock distribution. This removes uncertainty about second-period earnings and the earnings base for retirement income without changing their expected levels conditional on first-period earnings capacity.

We consider two experiments. In the *parents-only* experiment, the current generation faces no earnings shock, but descendants retain the baseline earnings-risk environment. We there-

fore solve the current household's problem using baseline descendants' continuation values. In the *all-generations* experiment, earnings risk is removed for both current parents and their descendants, and we re-solve the dynastic recursion.

Both experiments hold prices, mortality parameters, and all other calibrated parameters fixed. Outcomes are evaluated using the baseline distribution of initial household states; we do not recompute the stationary distribution. These are mechanism experiments rather than historical counterfactuals or fully specified insurance reforms.

Specifically, let μ_0 denote the baseline distribution of initial states and realized types over which outcomes are averaged. For environment q, let n^q denote fertility, s^q education spending per child, and a_k^q financial transfers per child. Mean fertility and child-weighted investments are

$$\begin{aligned} \bar{n}^q &= \int n^q \, d\mu_0, \\ \bar{s}^q_{\text{child}} &= \frac{\int n^q s^q \, d\mu_0}{\int n^q \, d\mu_0}, \qquad \bar{a}^q_{\text{child}} = \frac{\int n^q a_k^q \, d\mu_0}{\int n^q \, d\mu_0}. \end{aligned} \tag{24}$$

The same environment-specific fertility choices enter the numerator and denominator of each per-child measure. Changes in average investment therefore reflect both investment responses and changes in the distribution of children across families. We use this evaluation procedure for both the earnings-risk experiments and the historical mortality exercises in Section 6.

For VSL comparisons, we instead hold each current parent's fertility at its baseline choice while reoptimizing saving, education, and transfers conditional on that fertility. This separates changes in survival valuation at a given family size from changes associated with fertility adjustment. DVSL continues to value the common young-adult survival perturbation defined in Section 2.6.

5.2 Family choices

Table 6 reports the effects on family choices. Removing earnings risk for current parents alone raises completed fertility by 0.9%, with almost no change in education or transfers per child. When risk removal extends to descendants, fertility rises by 2.7%. Education per child falls by 3.6%, while financial transfers remain nearly unchanged, leaving total investment per child 2.8% below the baseline.

Table 6: Family responses to removing earnings risk

	Baseline	Parents only	All generations
Completed fertility	2.31	2.33	2.37
		(+0.9%)	(+2.7%)
Investment per child			
Education	0.0305	0.0305	0.0294
		(−0.03%)	(−3.6%)
Inter-vivos transfers	0.00855	0.00855	0.00854
		(−0.01%)	(−0.1%)
Total	0.039	0.039	0.038
		(−0.03%)	(−2.8%)

Notes: Earnings risk is removed for current parents only or for all generations, preserving the baseline mean earnings multiplier. Decision rules are reoptimized and evaluated on the baseline distribution of initial household states. Family outcomes use counterfactual fertility, and per-child investments are child-weighted. Investments are reported in model units. Figures in parentheses report percentage changes relative to the baseline.

The larger fertility response when risk removal extends to descendants reflects the improvement in their continuation values, which raises the benefit of parenthood. Aggregate investment responses also depend on which families expand. In the all-generations experiment, households retaining the same fertility increase financial transfers per child by 2.9% and reduce education spending by 1.6%, leaving total investment per child 0.6% lower. Fertility adjusters account for 4.1% of households, but their share of children rises from 2.3% to 4.9%. Reallocation toward these families accounts for approximately half of the aggregate decline in investment per child; changes within the two groups account for the remainder. Appendix A.3 reports the detailed decomposition.

5.3 Survival valuation

Table 7 distinguishes the own-life and dynastic effects of earnings uncertainty. Removing parents' risk raises $LVSL_1$ by 8.5%. Higher continuation utility and higher current consumption both contribute: the former raises the utility benefit of survival, while the latter lowers the marginal utility of the resources used to pay for it. The dynastic premium rises by only 0.3%, so $DVSL_1$ increases by a smaller 6.0%. Descendants' earnings-risk environment is unchanged in this experiment. The premium can nevertheless respond through current consumption and the inherited child state, but these channels produce only a small

Table 7: Survival valuation after removing earnings risk conditional on baseline fertility

	Baseline	Parents only	All generations
Life-cycle VSL, $LVSL_1$	2,061	2,236	2,245
		(+8.5%)	(+8.9%)
Dynastic VSL, $DVSL_1$	2,977	3,154	3,191
		(+6.0%)	(+7.2%)
Dynastic premium	916	918	946
		(+0.3%)	(+3.3%)
Components of the dynastic premium			
Continuation	719	721	751
		(+0.2%)	(+4.4%)
Inter-vivos transfers	38	39	39
		(+0.6%)	(+2.4%)
Human capital	158	159	156
		(+0.3%)	(−1.4%)

Notes: Household-weighted means in thousands of 2000 dollars. Both experiments use the baseline initial-state distribution and condition on each current household's baseline fertility, with other current choices reoptimized. $DVSL_1$ values the common young-adult survival perturbation, not the removal of earnings risk itself. Dollar values use the baseline income conversion in every environment. Figures in parentheses report percentage changes relative to the baseline.

net change in its population mean.

Removing risk across generations raises $LVSL_1$ by 8.9%, similar to the parents-only experiment, but produces a larger dynastic response. The premium increases by 3.3%, and $DVSL_1$ rises by 7.2%. The increase in the premium is dominated by its continuation component, which rises by 4.4%. The financial-transfer contribution also increases, while the human-capital contribution declines slightly. Thus, the larger premium does not require higher education spending per child. Reduced uncertainty raises descendants' continuation welfare and the value parents assign to their survival even at the reference inherited state without the current parent's discretionary investments.

These comparisons distinguish two channels through which earnings risk affects survival valuation. Removing current parents' risk mainly raises the own-life component, whereas extending risk removal to descendants also increases the dynastic premium. The premium nevertheless grows more slowly than the own-life component in both experiments, so its share of DVSL declines despite increasing in level. The generational reach of economic

uncertainty therefore matters for the composition as well as the level of survival valuation.[21]

6 Historical Mortality Improvements

Declining maternal mortality reduces a direct cost of childbearing and may therefore alter fertility and parental investment. Previous studies link improvements in maternal health to changes in family formation and women's economic choices (Jayachandran and Lleras-Muney 2009; Albanesi and Olivetti 2016).[22] On the other hand, mortality improvements later in life provide a different channel. Although they do not directly reduce the survival cost of childbirth, they change parents' life-cycle plans and the expected lifetimes of their descendants. In a dynastic framework, greater longevity can affect both the value of having children and the incentives to invest in their human capital (Hazan and Zoabi 2006).

We compare these channels through two historical experiments. Starting from the calibrated economy, we separately update maternal mortality and old-age survival from their 1940 values to their values one model period (20 years) later. Holding the remaining environment fixed allows us to examine how the timing of mortality improvements shapes family choices and the value parents assign to survival.

6.1 Design of the exercises

Both exercises start from the same baseline and change one mortality parameter at a time. In the maternal-mortality exercise, we reduce κ from 0.00377 to 0.00037, corresponding to the dramatic decline in maternal mortality from 376 to 37 deaths per 100,000 live births documented by Albanesi and Olivetti (2016). We hold $\tilde{\pi}_1$, π_2, and π_3 fixed. Survival during the childbearing period therefore improves through the reduction in the mortality risk associated with each birth.

In the older-age-survival exercise, we increase π_3 from 0.3701 to 0.507, using the corresponding female period life-table survival probabilities in 1940 and 1960. This change reflects the extension of U.S. mortality improvements to older ages documented by Cutler

[21] Appendix A.4 reports the welfare gains from removing earnings risk, expressed as equivalent one-time asset transfers in the baseline economy.

[22] For a historical account of maternal care and mortality, see Loudon (1992).

and Meara (2004). All other mortality parameters remain fixed. This change increases the probability of reaching the final period of life without directly changing the survival cost of childbirth.

In each exercise, households anticipate that the new mortality environment applies to themselves and subsequent generations. We re-solve the value functions and decision rules, taking prices and all non-mortality parameters as given. We then evaluate family choices on the baseline distribution of initial household states. The exercises therefore isolate responses to mortality improvements conditional on the inherited distribution; they do not describe the transition to a new stationary distribution.

Family outcomes use counterfactual policy rules and the baseline distribution, following equation (24). As in the earnings-risk experiments, the per-child investment measures incorporate changes in both investment decisions and the distribution of children across families. Valuation calculations likewise follow the convention in Section 5.1: current fertility remains at its baseline choice, while saving, education spending, and transfers are reoptimized. Appendix A.6 reports valuations at counterfactual fertility.

In both environments, $DVSL_1$ continues to price the same local permanent perturbation h to first-period survival defined in Section 2.6. In particular, the older-age exercise asks how higher π_3 changes the value of a marginal first-period survival improvement. Neither exercise reports total willingness to pay for the discrete historical change in κ or π_3.

6.2 Fertility and parental investment

Table 8 reports family outcomes. Both mortality improvements raise fertility and reduce childlessness. Lower maternal mortality reduces average education spending per child, while financial transfers change little, leaving total investment per child 1.1% below the baseline. Improved old-age survival slightly increases education spending and reduces financial transfers, leaving total investment per child approximately unchanged.

Lower maternal mortality raises completed fertility by 2.3%, from 2.31 to 2.36 children, with 3.5% of households increasing fertility. Reducing κ directly lowers the survival cost of childbirth, making parenthood and additional births more attractive. The resulting increase in family size is accompanied by a modest decline in average investment per child.

Table 8: Family responses to historical mortality improvements

	Baseline	Lower maternal mortality	Higher old-age survival
Completed fertility	2.31	2.36	2.34
		(+2.3%)	(+1.2%)
Childlessness rate	20.2%	18.7%	19.3%
Households changing fertility	—	3.5%	1.7%
Investment per child			
Education	0.0305	0.0301	0.0306
		(−1.4%)	(+0.4%)
Inter-vivos transfers	0.00855	0.00857	0.00843
		(+0.2%)	(−1.5%)
Total	0.03905	0.03864	0.03904
		(−1.1%)	(0.0%)
Investment per couple			
Education	0.07042	0.07103	0.07154
		(+0.9%)	(+1.6%)
Inter-vivos transfers	0.01975	0.02025	0.01969
		(+2.5%)	(−0.3%)

Notes: Decision rules are reoptimized in each environment and evaluated on the baseline distribution of initial household states. Per-child investments are child-weighted as in equation (24). Per-couple expenditures are $\int n^q s^q \, d\mu_0$ and $\int n^q a_k^q \, d\mu_0$; each adult finances one-half of these amounts. Investments are reported in model units. Figures in parentheses report percentage changes relative to the baseline.

Higher old-age survival raises fertility by 1.2%, from 2.31 to 2.34 children, with 1.7% of households increasing fertility. Unlike the maternal-mortality change, this improvement does not directly reduce the survival cost of childbirth. It instead changes parents' life-cycle plans and the continuation values of their children's dynasties. In the calibrated economy, these forces increase fertility while leaving average total investment per child approximately unchanged.

Households with unchanged fertility. Aggregate investment responses combine changes in parental investment decisions with changes in family size and the distribution of children across households. To distinguish these margins, Table 9 separately reports outcomes for households whose fertility remains unchanged.

Among households retaining the same fertility, total investment per child rises by 0.7% under lower maternal mortality and by 0.8% under higher old-age survival. The composition

Table 9: Changes in investment per child by fertility response

	Lower maternal mortality		Higher old-age survival	
	Full population	Fertility non-adjusters	Full population	Fertility non-adjusters
Education	-1.4%	$+0.1\%$	$+0.4\%$	$+1.2\%$
Inter-vivos transfers	$+0.2\%$	$+2.7\%$	-1.5%	-0.3%
Total investment	-1.1%	$+0.7\%$	0.0%	$+0.8\%$

Notes: Entries are percentage changes in child-weighted investment per child. Full-population columns allow both fertility choices and child weights to change. Non-adjuster columns restrict the calculation to households choosing the same fertility in both environments. These households account for 96.5% and 98.3% of households in the maternal-mortality and older-age-survival exercises, respectively. Household shares differ from the child shares entering per-child averages. Both environments use the baseline initial-state distribution. Childless households receive zero weight.

of this response differs: financial transfers account for most of the increase under lower maternal mortality, whereas education spending rises under higher old-age survival despite a slight decline in transfers. The weaker full-population responses therefore reflect more than investment adjustments at a given family size. Households increasing fertility have lower average investment per child in the counterfactual and account for a larger share of children. Their responses and the associated reallocation of child weights offset the increases among non-adjusters. Appendix A.7 reports the detailed results.

6.3 Changes in the value of survival

Table 10 reports survival valuations and the Shapley decomposition of the dynastic premium. These calculations hold the current parent's fertility at its baseline value, as described above.

Lower maternal mortality. Lower maternal mortality reduces both $LVSL_1$ and $DVSL_1$ by approximately 0.3%. The dynastic premium remains almost unchanged, declining by 0.1% from approximately \$915,800 to \$915,000.

These small declines do not imply a welfare loss. VSL measures marginal willingness to pay for an additional survival improvement in the new environment, rather than the welfare gain from the historical mortality change. Recall that

$$LVSL_1 = \beta \mathbb{E}_z[V_2] c_1^{\sigma}. \tag{25}$$

Table 10: Survival valuation across mortality environments conditional on baseline fertility

	Baseline	Lower maternal mortality	Higher old-age survival
Life-cycle VSL, $LVSL_1$	2,061	2,054	2,090
		(−0.3%)	(+1.4%)
Dynastic VSL, $DVSL_1$	2,977	2,969	3,025
		(−0.3%)	(+1.6%)
Dynastic premium	916	915	935
		(−0.1%)	(+2.1%)
Premium / $DVSL_1$	30.8%	30.8%	30.9%
Components of the dynastic premium			
Continuation	719	714	732
		(−0.7%)	(+1.8%)
Inter-vivos transfers	38	40	40
		(+4.8%)	(+3.6%)
Human capital	158	161	163
		(+1.4%)	(+3.1%)

Notes: Household-weighted means. Valuation levels are in thousands of 2000 dollars. Value functions are recomputed in each mortality environment and evaluated on the baseline state distribution and at baseline fertility. All other current choices are reoptimized conditional on baseline fertility. $DVSL_1$ values the local common first-period survival perturbation h, not the full historical mortality change. The premium is decomposed using the procedure in Section 4. Figures in parentheses report percentage changes relative to the baseline.

Lower κ directly increases the probability of reaching period 2 but does not directly change own-life utility at a given period-2 state. The conditional continuation value changes little, while first-period consumption falls slightly. The resulting increase in marginal utility lowers the monetary valuation of an additional survival improvement, outweighing the accompanying change in continuation utility.[23]

The nearly unchanged dynastic premium masks offsetting movements in its components. The continuation component falls by 0.7%, while the financial-transfer and human-capital contributions rise by 4.8% and 1.4%, respectively. Improved maternal survival therefore changes family choices without necessarily increasing the marginal dynastic valuation of a further common survival improvement.

[23] Bauer, Lakdawalla, and Reif (2025) show that an adverse longevity shock can raise VSL because the resulting increase in consumption can outweigh the reduction in remaining lifetime utility. Their analysis similarly highlights the role of consumption adjustments in monetary survival valuation.

Higher old-age survival. Higher π_3 raises $LVSL_1$ by 1.4% and $DVSL_1$ by 1.6%. The dynastic premium increases by 2.1%, from approximately \$0.92 million to \$0.93 million.

Unlike lower maternal mortality, higher old-age survival increases the weight placed on terminal-period utility within V_2, raising the value of reaching period 2 at a given asset position. First-period consumption also falls as parents adjust their life-cycle plans. The continuation-value effect outweighs the consumption-conversion effect, so mean $LVSL_1$ increases.

The dynastic premium also rises because the improvement extends to descendants. Its continuation component increases by 1.8% and accounts for approximately 67% of the increase in the premium. The financial-transfer and human-capital contributions rise by 3.6% and 3.1%, respectively. Most of the increase thus remains when descendants' survival benefits are evaluated at the reference inherited state without the current parent's discretionary investments.

The human-capital contribution should not be interpreted as a direct measure of education spending. It measures how the inherited human capital created by the current parent's education choice affects descendants' marginal survival valuation. Moreover, the decomposition conditions on baseline fertility, whereas the family outcomes in Table 8 allow fertility to adjust. Changes in the Shapley contribution therefore need not coincide with changes in education spending per child.

Valuation with fertility adjustment. The preceding results condition on current fertility at its baseline choice. Appendix A.6 instead evaluates survival valuations at each household's counterfactual fertility. Although only 3.5% and 1.7% of households change fertility in the two exercises, their adjustments affect both the level and composition of survival valuation.

Under lower maternal mortality, mean $LVSL_1$ falls by 2.8% at counterfactual fertility, compared with 0.3% at baseline fertility. The decline in $DVSL_1$ consequently becomes larger, at 1.8% rather than 0.3%. Under higher old-age survival, $LVSL_1$ rises by only 0.2%, compared with 1.4% at baseline fertility, while the increase in $DVSL_1$ falls from 1.6% to 0.9%.

Fertility adjustment does not similarly weaken the dynastic premium. At counterfactual fertility, the premium rises by 0.5% under lower maternal mortality and by 2.5% under higher old-age survival, compared with changes of -0.1% and $+2.1\%$ at baseline fertility.

Households entering parenthood acquire a positive dynastic component, while the resource demands of larger families can reduce current consumption and own-life valuation. Endogenous fertility therefore makes the response of total DVSL less positive without necessarily reducing its dynastic component.

7 Sensitivity to Consumption Curvature

Consumption curvature governs two margins relevant to our analysis: the intertemporal elasticity of substitution, $1/\sigma$, shapes consumption smoothing over time, while relative risk aversion, σ, shapes responses to uncertainty. Both can affect family choices and survival valuation. We assess sensitivity to consumption curvature by lowering σ from 1.5 to 1.25 and jointly recalibrating the eleven internally determined parameters to the same targets. All other externally set parameters remain unchanged. Each specification has its own stationary baseline distribution, which is held fixed within its counterfactual experiments. Appendix A.8 reports parameter estimates, calibration fit, and detailed experimental results.

Table 11 shows that the central findings concerning the dynastic premium remain similar. The premium accounts for 30.8% of young parents' total survival valuation in the baseline and 32.3% under lower consumption curvature. Its absolute level is approximately \$0.92 million in both calibrations, and continuation accounts for approximately four-fifths of the premium. Removing current parents' earnings risk alone leaves the premium almost unchanged, whereas removing risk across generations raises it by 3.3% and 3.7%, respectively. Lower maternal mortality also leaves the premium nearly unchanged in both specifications. Improved old-age survival raises it by approximately 2%, with continuation accounting for most of the increase.

Family responses are more sensitive to the recalibration. Under $\sigma = 1.25$, lower maternal mortality raises fertility by 3.8%, compared with 2.3% in the baseline, and removing earnings risk across generations raises fertility by 6.0%, compared with 2.7%. Both experiments produce larger declines in aggregate investment per child. Improved old-age survival nevertheless leaves total investment per child approximately unchanged in both economies. Among historical fertility non-adjusters, total investment per child continues to rise: by 0.8% under lower maternal mortality and 1.1% under higher old-age survival when $\sigma = 1.25$. The distinction between investment responses at unchanged fertility and full-population out-

Table 11: Sensitivity to consumption curvature

	$\sigma = 1.5$	$\sigma = 1.25$
Baseline valuation shares (%)		
Dynastic premium / DVSL	30.8	32.3
Continuation / dynastic premium	78.5	79.5
Change in dynastic premium (%)		
Removing earnings risk: parents only	$+0.3$	$+0.2$
Removing earnings risk: all generations	$+3.3$	$+3.7$
Lower maternal mortality	-0.1	$+0.2$
Higher old-age survival	$+2.1$	$+2.2$
Change in total investment per child (%)		
Removing earnings risk: all generations	-2.8	-7.2
Lower maternal mortality	-1.1	-4.4
Higher old-age survival	≈ 0	$+0.04$

Notes: Each specification is calibrated to the same targets. Experimental changes are relative to the corresponding baseline and evaluated on its initial-state distribution. Investment outcomes use counterfactual fertility and child weights. Valuations condition on current parents' baseline fertility, with other current choices reoptimized.

comes therefore remains.

The quantitative robustness is stronger for the overall dynastic premium than for individual investment margins or all of its Shapley components. For example, the human-capital contribution under all-generations risk removal declines slightly in the baseline but increases under lower consumption curvature. Continuation remains the dominant source of the premium increase in both economies.

8 Conclusion

We develop a quantitative dynastic life-cycle model to study how fertility and parental investments shape survival valuation. In the calibrated economy, the dynastic premium accounts for approximately 31% of young adults' dynastic VSL. Most reflects dynastic continuation, while education spending and financial transfers increase it further. Removing earnings risk across generations produces larger fertility and dynastic-premium responses than removing current parents' risk alone, highlighting the importance of descendants' economic prospects.

The historical exercises show that mortality improvements reshape the quantity-quality trade-off even with fixed wages and human-capital technology. Both improvements raise fertility, but fertility adjustment and changes in child weights weaken the aggregate investment response relative to families maintaining fertility. At baseline fertility, the dynastic premium changes little under lower maternal mortality but rises under improved old-age survival. These differences highlight the importance of which stages of life benefit from mortality improvements.

These findings connect the quantity-quality and dynastic fertility tradition (Becker and Lewis 1973; Barro and Becker 1989) with the life-cycle valuation of mortality improvements (Murphy and Topel 2006; Hall and Jones 2007). Their broader implication is that survival valuation depends not only on how long people live, but also on the economic circumstances and family decisions connecting generations. Our DVSL includes descendants' survival benefits as valued by the current parent, alongside the parent's own-life valuation, rather than summing welfare gains across generations. The framework therefore informs comparisons of mortality improvements benefiting different ages and generations, without prescribing a uniform dynastic adjustment to empirical VSL estimates.

We take mortality parameters as given to isolate their effects on family choices and survival valuation. An extension with health investment and health-dependent utility would introduce an additional allocation margin between parental health and investments in children. It would also allow us to value improvements in both the length and health-related quality of life, and examine how family resources and decisions shape these outcomes across generations.

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A Online Appendix

A.1 Distributional Dynamics and Stationary Equilibrium

We characterize a stationary equilibrium where household decisions determine the distribution of inherited resources across generations and the distribution of surviving agents over the life cycle. Stationarity requires these normalized distributions to remain unchanged, although population size may grow or decline.

Distribution across generations. Let $x = (a, e, \phi, \alpha)$ denote a young agent's state, and let μ_t denote its distribution among households entering adulthood at date t, normalized to unit mass. Write the optimal fertility choice as $n(x)$. Conditional on child ability ϕ', parental transfers and education spending determine the child's inherited state:

$$a' = a_k(x, \phi'), \qquad e' = A\phi' \big[\theta + s(x, \phi')\big]^{\nu}.$$

Child ability follows the intergenerational transition specified above, while the child's altruism is drawn independently from $F_\alpha(\alpha)$. These transitions and the parental investment policies induce a probability kernel $K(B \mid x)$, giving the probability that a child of a household in state x enters adulthood with a state in the set B.

The distribution of entering households evolves according to

$$\mu_{t+1}(B) = \frac{\int n(x) K(B \mid x)\, \mu_t(dx)}{\int n(x)\, \mu_t(dx)}. \tag{A1}$$

Thus, households with more children contribute proportionately more to the next generation, while childless households do not contribute to its distribution. Because fertility n counts children per couple, the gross growth factor of entering adult cohorts is

$$G_t = \frac{1}{2} \int n(x)\, \mu_t(dx). \tag{A2}$$

The factor of one-half cancels from the normalized distribution in equation (A1). A stationary distribution μ is a fixed point of this law of motion, with associated cohort growth factor G.

Distribution over the life cycle. Let $Q_j(B \mid x)$ denote the transition probability to a set of age-$(j+1)$ states, conditional on survival, induced by optimal saving and the exogenous shocks. Survival is $\pi_1(x) = \tilde{\pi}_1 \exp[-\kappa n(x)]$ at the first age and π_j at ages $j = 2, 3$. Starting from the entering-cohort measure $m_1 = \mu$, the stationary age-specific measures satisfy

$$m_{j+1}(B) = \frac{1}{G} \int \pi_j(x) Q_j(B \mid x)\, m_j(dx), \qquad j = 1, 2, 3. \tag{A3}$$

The survival probability accounts for mortality, while division by G accounts for differences in the sizes of successive entering cohorts. The cross-sectional distribution is obtained by normalizing these measures:

$$\Lambda_j(B) = \frac{m_j(B)}{\sum_{\ell=1}^{4} m_\ell(\mathcal{X}_\ell)}, \qquad j = 1, \ldots, 4, \tag{A4}$$

where $\mathcal{X}_\ell$ is the state space at age ℓ.

A stationary equilibrium consists of household value and policy functions, an entering-cohort distribution μ, a cohort growth factor G, and age-specific cross-sectional measures $\{\Lambda_j\}_{j=1}^{4}$ such that: households optimize given prices, mortality risks, and descendants' value functions; μ and G satisfy equations (A1) and (A2); and the cross-sectional distribution satisfies equations (A3) and (A4).

A.2 Construction of the dynastic-premium decomposition

Consider an agent at its baseline choices and realized types. Holding the current parent's fertility n^*, consumption c_1^*, altruism α, and child ability ϕ_k fixed, define

$$\mathcal{D}(a, e) \equiv \alpha g(n^*)(c_1^*)^\sigma \exp(\kappa n^*) \left. \frac{\partial W'(a, e, \phi_k; h)}{\partial h} \right|_{h=0}. \tag{A5}$$

This function evaluates the dynastic premium when the child inherits assets a and earnings capacity e. The child and subsequent generations optimize conditional on the inherited state; the current parent's choices remain fixed.

Define earnings capacity without discretionary parental education spending and under base-

line spending, respectively, as

$$e_k^0 = A\phi_k\theta^\nu, \qquad e_k^* = A\phi_k(\theta + s^*)^\nu. \tag{A6}$$

We evaluate the premium at four inherited states:

$$\begin{aligned} D_{00} &= \mathcal{D}(0, e_k^0), & D_{10} &= \mathcal{D}(a_k^*, e_k^0), \\ D_{01} &= \mathcal{D}(0, e_k^*), & D_{11} &= \mathcal{D}(a_k^*, e_k^*). \end{aligned} \tag{A7}$$

The actual baseline inherited state satisfies

$$D_{11} = DVSL_1 - LVSL_1. \tag{A8}$$

The continuation component is $C = D_{00}$. The Shapley contributions average the marginal effect of each inherited resource over the two possible orderings:

$$S_a = \frac{1}{2}\left[(D_{10} - D_{00}) + (D_{11} - D_{01})\right], \tag{A9}$$

$$S_e = \frac{1}{2}\left[(D_{01} - D_{00}) + (D_{11} - D_{10})\right]. \tag{A10}$$

These definitions imply the exact accounting identity in equation (22).

A.3 Earnings Risk and Investment Responses by Fertility Adjustment

Table A1 separates investment responses by whether households change fertility. Groups are defined separately for each experiment, and both environments are evaluated on the baseline initial-state distribution.

Removing current parents' earnings risk changes fertility for only 0.9% of households. Investment per child rises slightly among non-adjusters, while the full-population average remains almost unchanged. When risk removal extends to descendants, 4.1% of households increase fertility, and their share of children rises from 2.3% to 4.9%.

Among non-adjusters in the all-generations experiment, education spending per child falls by 1.6% and transfers rise by 2.9%, leaving total investment 0.6% lower. The full-population decline of 2.8% also reflects lower investment among adjusters and the increasing share of

Table A1: Investment responses to removing earnings risk by fertility adjustment

	Household share	Share of children		Investment change (%)		
		Baseline	Counterfactual	Education	Transfers	Total
A. Parents-only risk removal						
Non-adjusters	99.1%	99.8%	98.9%	+0.2	+0.5	+0.3
Adjusters	0.9%	0.2%	1.1%	—	—	—
All households	100%	100%	100%	−0.03	−0.01	−0.03
B. All-generations risk removal						
Non-adjusters	95.9%	97.7%	95.1%	−1.6	+2.9	−0.6
Adjusters	4.1%	2.3%	4.9%	−40.1	−61.6	−43.9
All households	100%	100%	100%	−3.6	−0.1	−2.8

Notes: Investment entries are percentage changes in child-weighted investment per child. Household groups are defined by whether fertility changes in the relevant experiment. Child shares are the weights entering full-population per-child averages in each environment. Adjuster entries compare group-level child-weighted averages, not average household-level percentage changes. Investment changes for parents-only adjusters are omitted because this group accounts for a very small baseline share of children. Both environments use the baseline distribution of initial household states.

children in these families. Reallocation accounts for approximately half of the aggregate decline.

A.4 Welfare gains from removing earnings risk

The results in Section 5 concern the value of a marginal survival improvement in different earnings-risk environments. Here we measure the welfare gain from removing earnings risk itself. This complementary calculation distinguishes benefits confined to current parents from those extending to descendants.

We express the gain as a uniform one-time addition T to current young parents' initial assets, measured per adult, in the baseline economy that reproduces average counterfactual dynastic welfare:

$$\int W^0(a_1 + T, e_1, \phi)\, d\mu_1^0(a_1, e_1, \phi) = \int W^{CF}(a_1, e_1, \phi)\, d\mu_1^0(a_1, e_1, \phi), \tag{A11}$$

where μ_1^0 is the baseline distribution of young households' inherited states before the altruism draw, W^0 is the baseline ex-ante dynastic value, and W^{CF} is its counterpart under

Table A2: Welfare gains from removing earnings risk

	Parents only	All generations
Equivalent asset transfer, T (\$1,000)	8.9	15.5
T / baseline average income (%)	1.26	2.20

Notes: Transfers are expressed in 2000 dollars. Average income refers to one 20-year model period. The uniform asset transfer matches average counterfactual dynastic welfare on the baseline initial-state distribution, as defined in equation (A11).

the relevant risk-removal experiment. These value functions integrate over altruism draws. Unlike the VSL comparisons, the welfare calculation allows fertility to adjust, including in response to the compensating asset transfer. The same transfer is provided to every young household and is chosen to match average welfare.

Table A2 reports an equivalent asset transfer of approximately \$8,900 when risk is removed for current parents alone and \$15,500 when risk removal extends to descendants. The all-generations transfer equivalent is about 1.7 times as large. This additional value reflects descendants' improved economic circumstances as valued by current parents, rather than a sum of separately measured welfare gains across generations. Dynastic links therefore matter for valuing safer economic lives as well as longer lives.

A.5 Consumption and Continuation Values in Life-Cycle VSL

The life-cycle value of survival depends on both the utility gain from survival and the marginal utility of current consumption. Define

$$B_j \equiv \beta \mathbb{E}_j[V_{j+1}]. \tag{A12}$$

Then

$$LVSL_j = c_j^{\sigma} B_j. \tag{A13}$$

For households with positive consumption and $B_j > 0$,

$$\ln LVSL_j = \sigma \ln c_j + \ln B_j. \tag{A14}$$

Averaging this identity over the same households separates the consumption and continuation-value components of mean log LVSL. With common weights and sample coverage across

Table A3: Consumption and continuation-value components of LVSL

Age	Environment	Mean $\ln c_j$	Mean $\ln B_j$	Mean $\ln LVSL_j$
$j = 1$	Baseline	-0.9984	2.0544	0.5569
	Lower maternal mortality	-1.0016	2.0545	0.5521
	Higher old-age survival	-1.0040	2.0776	0.5715
$j = 2$	Baseline	-0.1279	1.7788	1.5869
	Higher old-age survival	-0.1459	1.8435	1.6246
$j = 3$	Baseline	-0.0879	1.5131	1.3813
	Higher old-age survival	-0.1366	1.5433	1.3385

Notes: Logarithms are taken in raw model units before the common conversion into dollars. Each environment uses baseline fertility and baseline state weights. Within each row,

$$\text{mean}(\ln LVSL_j) = \sigma \,\text{mean}(\ln c_j) + \text{mean}(\ln B_j),$$

up to rounding, with $\sigma = 1.5$. Changes in mean log LVSL need not equal percentage changes in arithmetic mean LVSL. Older-age results under lower maternal mortality are omitted because they coincide with the baseline.

environments, changes in mean log LVSL can therefore be decomposed into changes in these two components.

Table A3 reports log means for the baseline-fertility evaluation used in the main text. Under lower maternal mortality, the age-1 continuation-value component is essentially unchanged. Mean log consumption falls by approximately 0.0032, contributing approximately -0.0048 to the change in mean log $LVSL_1$.

Under improved old-age survival, continuation values rise while consumption falls. At age 1, the continuation-value contribution is approximately $+0.0232$, compared with a consumption contribution of -0.0084. The continuation-value effect also dominates at age 2. At age 3, its contribution of approximately $+0.0302$ is outweighed by a consumption contribution of approximately -0.0731, so mean log $LVSL_3$ falls.

A.6 Survival Valuation with Fertility Adjustment

The main-text valuation results evaluate each counterfactual environment at the current household's baseline fertility. Here we instead evaluate survival valuation at its counterfactual fertility choice. Both calculations use the baseline distribution of initial household

states. Comparing them shows how endogenous family-size adjustment changes marginal willingness to pay for survival.

Table A4: Survival valuations at counterfactual fertility

Household group	Measure	Baseline	Counterfactual	Change (%)
A. Lower maternal mortality				
Non-adjusters (96.5%)	$LVSL_1$	2,045	2,038	-0.3
	$DVSL_1$	2,977	2,969	-0.3
	Dynastic premium	933	932	-0.1
Adjusters (3.5%)	$LVSL_1$	2,502	1,069	-57.3
	$DVSL_1$	2,950	1,676	-43.2
	Dynastic premium	447	607	$+35.7$
All households	$LVSL_1$	2,061	2,004	-2.8
	$DVSL_1$	2,977	2,924	-1.8
	Dynastic premium	916	920	$+0.5$
B. Higher old-age survival				
Non-adjusters (98.3%)	$LVSL_1$	2,054	2,083	$+1.4$
	$DVSL_1$	2,979	3,028	$+1.6$
	Dynastic premium	925	945	$+2.1$
Adjusters (1.7%)	$LVSL_1$	2,449	1,012	-58.7
	$DVSL_1$	2,806	1,584	-43.5
	Dynastic premium	357	572	$+60.2$
All households	$LVSL_1$	2,061	2,065	$+0.2$
	$DVSL_1$	2,977	3,003	$+0.9$
	Dynastic premium	916	938	$+2.5$

Notes: Household-weighted means in thousands of 2000 dollars. Baseline valuations use baseline fertility; counterfactual valuations use each household's counterfactual fertility. Both columns use the baseline initial-state distribution and the same baseline income conversion. Groups are defined by whether fertility changes in the corresponding experiment. Large percentage increases in adjusters' premiums partly reflect their small baseline premiums and entry into parenthood.

Table A4 separates households whose fertility remains unchanged from those whose fertility changes. Groups are defined separately for each experiment, and the same households are compared across environments within each group. For non-adjusters, the baseline-fertility and counterfactual-fertility evaluation conventions coincide.

Under lower maternal mortality, 3.5% of households increase fertility. Evaluated at their counterfactual choices, adjusters' mean $LVSL_1$ falls by 57.3% and $DVSL_1$ falls by 43.2%, while their dynastic premium rises by 35.7%. Under higher old-age survival, 1.7% increase

fertility; their mean $LVSL_1$ falls by 58.7% and $DVSL_1$ falls by 43.5%, while the premium rises by 60.2%. These group-level changes combine the responses of existing parents and households entering parenthood.

At the aggregate level, evaluating valuations at counterfactual fertility lowers the response of $DVSL_1$ from -0.3% to -1.8% under lower maternal mortality and from $+1.6\%$ to $+0.9\%$ under higher old-age survival. The premium instead rises by 0.5% and 2.5%, respectively, compared with -0.1% and $+2.1\%$ at baseline fertility. Fertility adjustment therefore lowers own-life valuation relative to the fixed-fertility comparison without eliminating the dynastic premium. Entry into parenthood creates a positive dynastic component, while the resource demands of larger families can reduce current consumption and own-life willingness to pay.

These calculations concern marginal willingness to pay for survival at the counterfactual allocation, not the welfare gains from the historical mortality improvements. Changes in consumption and its marginal utility can lower monetary survival valuations even when households benefit from the improved environment.

A.7 Fertility Adjusters and Investment per Child

Table A5: Fertility adjustment and investment per child

Mortality change	Household group	Education	Transfers	Total
Lower maternal mortality	Non-adjusters (96.5%)	$+0.1$	$+2.7$	$+0.7$
	Adjusters (3.5%)	-37.9	-59.9	-42.2
	All households	-1.4	$+0.2$	-1.1
Higher old-age survival	Non-adjusters (98.3%)	$+1.2$	-0.3	$+0.8$
	Adjusters (1.7%)	-38.8	-59.4	-42.1
	All households	$+0.4$	-1.5	0.0

Notes: Entries are percentage changes in child-weighted investment per child. Non-adjusters choose the same fertility in both environments; adjusters increase fertility. Parenthetical shares are household shares, not the child shares entering per-child averages. Comparisons allow child weights to change within each group. Both environments use the baseline initial-state distribution. Childless households receive zero weight.

Table A5 provides additional detail on investment responses by fertility adjustment. Groups are defined separately for each mortality experiment, and both environments are evaluated using the baseline distribution of initial household states.

Under lower maternal mortality, non-adjusters account for 96.5% of households and increase total investment per child by 0.7%. Among adjusters, education spending per child falls by 37.9% and transfers fall by 59.9%, producing a 42.2% decline in combined investment. Under higher old-age survival, non-adjusters account for 98.3% of households and increase total investment per child by 0.8%, while adjusters' combined investment falls by 42.1%.

These entries compare group-level child-weighted averages, rather than averaging individual parents' percentage changes. Adjusters include households entering parenthood as well as existing parents increasing family size. Their investment responses and their growing share of children weaken the full-population response relative to that of non-adjusters.

A.8 Alternative Consumption Curvature

This appendix reports the recalibration and results from the experiments for $\sigma = 1.25$. Table A6 compares parameter estimates and model moments. The calibration uses the same eleven targets as the baseline, including a \$2 million target for mean young-adult LVSL, corresponding to an income-normalized value of approximately 2.833.

The alternative calibration provides a reasonably close fit, although some moments differ from their targets. Mean young-adult LVSL is approximately \$1.93 million, the capital-output ratio is 0.151, and first-period log-income variance is 0.570, compared with targets of \$2 million, 0.150, and 0.550, respectively. Fertility levels are somewhat lower than their empirical counterparts: mean completed fertility is 2.17, compared with 2.30 in the data. The model nevertheless reproduces a bottom-top income fertility gap of approximately 0.54 children, close to the targeted gap of 0.57.

Table A7 compares the experimental responses. Prices and externally set parameters other than σ remain unchanged across specifications. Within each specification, experiments use its own baseline initial-state distribution and the valuation conventions described in Sections 5 and 6. Dollar conversions use each specification's baseline average income and are held fixed across its experiments.

Table A6: Alternative consumption curvature: parameters and calibration fit

A. Parameters

Parameter	$\sigma = 1.5$	$\sigma = 1.25$
b	10.78	11.42
μ_α	0.263	0.280
β	0.584	0.580
ρ_ϕ	0.242	0.204
σ_ϕ	0.461	0.425
σ_z	0.461	0.499
A	3.05	2.78
δ	4.40	3.99
θ	0.029	0.033
ϵ	0.671	0.770
σ_α	0.336	0.273

B. Model moments

Moment	Target	$\sigma = 1.5$	$\sigma = 1.25$
Young-adult LVSL ($ million)	2.00	2.06	1.93
Inter-vivos transfers / GDP	0.0200	0.0201	0.0196
Capital-output ratio	0.150	0.148	0.151
Intergenerational income elasticity	0.350	0.350	0.343
$\mathrm{Var}(\log y_1)$	0.550	0.548	0.570
$\mathrm{Var}(\log y_2) - \mathrm{Var}(\log y_1)$	0.180	0.180	0.181
Average income	1.000	0.985	0.994
Childlessness rate	0.203	0.202	0.199
Education expenditures / GDP	0.0730	0.0715	0.0737
Completed fertility, bottom income half	2.58	2.52	2.44
Completed fertility, top income half	2.01	2.10	1.91

Notes: The eleven internal parameters are recalibrated jointly using the same targets and equally weighted squared percentage deviations. Parameter definitions and target construction follow Section 3. VSL is expressed in millions of 2000 dollars.

Table A7: Computational experimental responses under alternative consumption curvature

A. Removing earnings risk	Parents only		All generations	
	$\sigma = 1.5$	$\sigma = 1.25$	$\sigma = 1.5$	$\sigma = 1.25$
Completed fertility	$+0.9$	$+0.4$	$+2.7$	$+6.0$
Education per child	-0.03	$+0.5$	-3.6	-6.4
Transfers per child	-0.01	$+0.8$	-0.1	-10.6
Total investment per child	-0.03	$+0.6$	-2.8	-7.2
$LVSL_1$	$+8.5$	$+6.8$	$+8.9$	$+7.0$
$DVSL_1$	$+6.0$	$+4.6$	$+7.2$	$+5.9$
Dynastic premium	$+0.3$	$+0.2$	$+3.3$	$+3.7$
Continuation	$+0.2$	$+0.2$	$+4.4$	$+4.0$
Financial transfers	$+0.6$	$+0.5$	$+2.4$	$+5.0$
Human capital	$+0.3$	$+0.3$	-1.4	$+2.0$
B. Historical mortality improvements	Lower maternal mortality		Higher old-age survival	
	$\sigma = 1.5$	$\sigma = 1.25$	$\sigma = 1.5$	$\sigma = 1.25$
Completed fertility	$+2.3$	$+3.8$	$+1.2$	$+1.6$
Education per child	-1.4	-4.1	$+0.4$	$+0.5$
Transfers per child	$+0.2$	-5.3	-1.5	-1.8
Total investment per child	-1.1	-4.4	≈ 0	$+0.04$
$LVSL_1$	-0.3	-0.4	$+1.4$	$+1.4$
$DVSL_1$	-0.3	-0.2	$+1.6$	$+1.7$
Dynastic premium	-0.1	$+0.2$	$+2.1$	$+2.2$
Continuation	-0.7	-0.4	$+1.8$	$+2.0$
Financial transfers	$+4.8$	$+6.8$	$+3.6$	$+3.4$
Human capital	$+1.4$	$+1.5$	$+3.1$	$+3.2$

Notes: All entries are percentage changes relative to the corresponding specification's baseline. Each experiment uses its specification's baseline initial-state distribution. Family outcomes use counterfactual fertility, and per-child investments are child-weighted. Survival valuations are household-weighted and condition on current parents' baseline fertility, with other current choices reoptimized. Component entries report percentage changes in each Shapley component, not contributions to the percentage change in the total premium. Each specification uses its own baseline income conversion, held fixed across its experiments.